OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4723

Core Mathematics 3

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

Registered Charity Number: 1066969
[Turn over

1 Find the equation of the tangent to the curve $y = \sqrt{4x + 1}$ at the point (2, 3). [5]

2 Solve the inequality $|2x - 3| < |x + 1|$. [5]

3 The equation $2x^3 + 4x - 35 = 0$ has one real root.

(i) Show by calculation that this real root lies between 2 and 3. [3]

(ii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{17.5 - 2x_n},$$

with a suitable starting value, to find the real root of the equation $2x^3 + 4x - 35 = 0$ correct to 2 decimal places. You should show the result of each iteration. [3]

4 It is given that $y = 5^{x-1}$.

(i) Show that $x = 1 + \dfrac{\ln y}{\ln 5}$. [2]

(ii) Find an expression for $\dfrac{dx}{dy}$ in terms of y. [2]

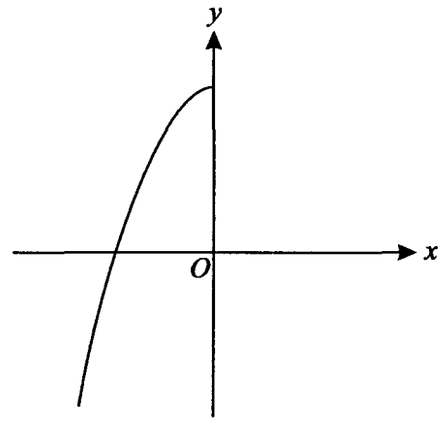
(iii) Hence find the exact value of the gradient of the curve $y = 5^{x-1}$ at the point (3, 25). [2]

5 **(i)** Write down the identity expressing $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. [1]

(ii) Given that $\sin\alpha = \frac{1}{4}$ and α is acute, show that $\sin 2\alpha = \frac{1}{8}\sqrt{15}$. [3]

(iii) Solve, for $0° < \beta < 90°$, the equation $5 \sin 2\beta \sec\beta = 3$. [3]

6



The diagram shows the graph of $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2 - x^2, \qquad x \leqslant 0.$$

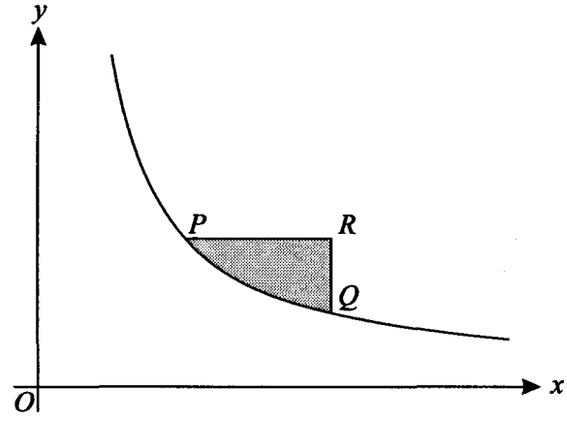
(i) Evaluate $\mathrm{ff}(-3)$. [3]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(iii) Sketch the graph of $y = \mathrm{f}^{-1}(x)$. Indicate the coordinates of the points where the graph meets the axes. [3]

7 **(a)** Find the exact value of $\displaystyle\int_1^2 \frac{2}{(4x-1)^2}\,\mathrm{d}x$. [4]

(b)



The diagram shows part of the curve $y = \frac{1}{x}$. The point P has coordinates $\left(a, \frac{1}{a}\right)$ and the point Q has coordinates $\left(2a, \frac{1}{2a}\right)$, where a is a positive constant. The point R is such that PR is parallel to the x-axis and QR is parallel to the y-axis. The region shaded in the diagram is bounded by the curve and by the lines PR and QR. Show that the area of this shaded region is $\ln\left(\frac{1}{2}\mathrm{e}\right)$. [6]

[Turn over

8 **(i)** Express $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence give details of a pair of transformations which transforms the curve $y = \cos x$ to the curve $y = 5\cos x + 12\sin x$. [3]

(iii) Solve, for $0° < x < 360°$, the equation $5\cos x + 12\sin x = 2$, giving your answers correct to the nearest $0.1°$. [5]

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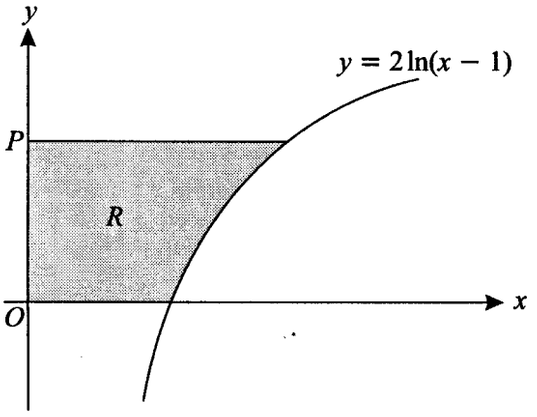

The diagram shows the curve with equation $y = 2\ln(x - 1)$. The point P has coordinates $(0, p)$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units on the axes are centimetres. The region R is rotated completely about the ***y*-axis** to form a solid.

(i) Show that the volume, V cm^3, of the solid is given by

$$V = \pi\left(e^p + 4e^{\frac{1}{2}p} + p - 5\right).$$ [8]

(ii) It is given that the point P is moving in the positive direction along the y-axis at a constant rate of 0.2 cm min^{-1}. Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures. [5]

4723 - Core Mathematics 3 - June 2006 - Mark Scheme

1	Differentiate to obtain $k(4x+1)^{-\frac{1}{2}}$		**M1**	any non-zero constant k
	Obtain $2(4x+1)^{-\frac{1}{2}}$	**A1**		or equiv, perhaps unsimplified
	Obtain $\frac{2}{3}$ for value of first derivative	**A1**		or unsimplified equiv
	Attempt equation of tangent through (2, 3)	**M1**		using numerical value of first derivative provided derivative is of form $k'(4x+1)^n$
	Obtain $y = \frac{2}{3}x + \frac{5}{3}$ or $2x - 3y + 5 = 0$	**A1**	**5**	or equiv involving 3 terms

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2	Either:	Attempt to square both sides	**M1**		producing 3 terms on each side
		Obtain $3x^2 - 14x + 8 = 0$	**A1**		or inequality involving < or >
		Obtain correct values $\frac{2}{3}$ and 4	**A1**		
		Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
		Obtain $\frac{2}{3} < x < 4$	**A1**	**5**	or correctly expressed equiv; allow ≤ signs
	Or:	Attempt solution of two linear equations or inequalities	**M1**		one eqn with signs of $2x$ and x the same, second eqn with signs different
		Obtain value $\frac{2}{3}$	**A1**		
		Obtain value 4	**B1**		
		Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
		Obtain $\frac{2}{3} < x < 4$	**A1**	**(5)**	or correctly expressed equiv; allow ≤ signs

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3	**(i)**	Attempt evaluation of cubic expression at 2 and 3	**M1**		
		Obtain –11 and 31	**A1**		
		Conclude by noting change of sign	**A1√**	**3**	or equiv; following any calculated values provided negative then positive
	(ii)	Obtain correct first iterate	**B1**		using x_1 value such that $2 \le x_1 \le 3$
		Attempt correct process to obtain at least 3 iterates	**M1**		using any starting value now
		Obtain 2.34	**A1**	**3**	answer required to 2 d.p. exactly; 2→2.3811→2.3354→2.3410; 2.5→2.3208→2.3428→2.3401; 3→2.2572→2.3505→2.3392

4	**(i)**	State $\ln y = (x-1)\ln 5$	**B1**		whether following $\ln y = \ln 5^{x-1}$ or not; brackets needed
		Obtain $x = 1 + \dfrac{\ln y}{\ln 5}$	**B1**	**2**	**AG**; correct working needed; missing brackets maybe now implied
	(ii)	Differentiate to obtain single term of form $\dfrac{k}{y}$	**M1**		any constant k
		Obtain $\dfrac{1}{y \ln 5}$	**A1**	**2**	or equiv involving y
	(iii)	Substitute for y and attempt reciprocal	**M1**		or equiv method for finding derivative without using part **(ii)**
		Obtain $25 \ln 5$	**A1**	**2**	or exact equiv
5	**(i)**	State $\sin 2\theta = 2 \sin\theta \cos\theta$	**B1**	**1**	or equiv; any letter acceptable here (and in parts **(ii)** and **(iii)**)
	(ii)	Attempt to find exact value of $\cos\alpha$	**M1**		using identity attempt or right-angled triangle
		Obtain $\frac{1}{4}\sqrt{15}$	**A1**		or exact equiv
		Substitute to confirm $\frac{1}{8}\sqrt{15}$	**A1**	**3**	**AG**
	(iii)	State or imply $\sec\beta = \dfrac{1}{\cos\beta}$	**B1**		
		Use identity to produce equation involving $\sin\beta$	**M1**		
		Obtain $\sin\beta = 0.3$ and hence 17.5	**A1**	**3**	and no other values between 0 and 90; allow 17.4 or value rounding to 17.4 or 17.5
6	**(i)**	Either: Obtain f(–3) = –7	**B1**		maybe implied
		Show correct process for compn of functions	**M1**		
		Obtain –47	**A1**	**3**	
		Or: Show correct process for compn of functions	**M1**		using algebraic approach
		Obtain $2 - (2 - x^2)^2$	**A1**		or equiv
		Obtain –47	**A1**	**(3)**	
	(ii)	Attempt correct process for finding inverse	**M1**		as far as $x = \ldots$ or equiv
		Obtain either one of $x = \pm\sqrt{2-y}$ or both	**A1**		or equiv perhaps involving x
		Obtain correct $-\sqrt{2-x}$	**A1**	**3**	or equiv; in terms of x now
	(iii)	Draw graph showing attempt at reflection in $y = x$	**M1**		
		Draw (more or less) correct graph	**A1**		with end-point on x-axis and no minimum point in third quadrant
		Indicate coordinates 2 and $-\sqrt{2}$	**A1**	**3**	accept –1.4 in place of $-\sqrt{2}$

7	**(a)**	Obtain integral of form $k(4x-1)^{-1}$	**M1**	any non-zero constant k
		Obtain $-\frac{1}{2}(4x-1)^{-1}$	**A1**	or equiv; allow + c
		Substitute limits and attempt evaluation	**M1**	for any expression of form $k'(4x-1)^n$
		Obtain $\frac{2}{21}$	**A1** **4**	or exact equiv
	(b)	Integrate to obtain $\ln x$	**B1**	
		Substitute limits to obtain $\ln 2a - \ln a$	**B1**	
		Subtract integral attempt from attempt at area of appropriate rectangle	**M1**	or equiv
		Obtain $1 - (\ln 2a - \ln a)$	**A1**	or equiv
		Show at least one relevant logarithm property	**M1**	at any stage of solution
		Obtain $1 - \ln 2$ and hence $\ln(\frac{1}{2}\mathrm{e})$	**A1** **6**	**AG**; full detail required

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8	**(i)**	State $R = 13$	**B1**	or equiv
		State at least one equation of form $R\cos\alpha = k$, $R\sin\alpha = k'$, $\tan\alpha = k''$	**M1**	or equiv; allow sin / cos muddles; implied by correct α
		Obtain 67.4	**A1** **3**	allow 67 or greater accuracy
	(ii)	Refer to translation and stretch	**M1**	in either order; allow here equiv terms such as 'move', 'shift'; with both transformations involving constants
		State translation in positive x direction by 67.4	**A1√**	or equiv; following their α; using correct terminology now
		State stretch in y direction by factor 13	**A1√** **3**	or equiv; following their R; using correct terminology now
	(iii)	Attempt value of $\cos^{-1}(2 \div R)$	**M1**	
		Obtain 81.15	**A1√**	following their R; accept 81
		Obtain 148.5 as one solution	**A1**	accept 148.5 or 148.6 or value rounding to either of these
		Add their α value to second value correctly attempted	**M1**	
		Obtain 346.2	**A1** **5**	accept 346.2 or 346.3 or value rounding to either of these; and no other solutions

9	**(i)**	Attempt to express x in terms of y	***M1**	obtaining two terms
		Obtain $x = \mathrm{e}^{\frac{1}{2}y} + 1$	**A1**	or equiv
		State or imply volume involves $\int \pi x^2$	**B1**	
		Attempt to express x^2 in terms of y	***M1**	dep ***M**; expanding to produce at least 3 terms
		Obtain $k\int (\mathrm{e}^{y} + 2\mathrm{e}^{\frac{1}{2}y} + 1)\,\mathrm{d}y$	**A1**	any constant k including 1; allow if dy absent
		Integrate to obtain $k(\mathrm{e}^{y} + 4\mathrm{e}^{\frac{1}{2}y} + y)$	**A1**	
		Use limits 0 and p	**M1**	dep ***M *M**; evidence of use of 0 needed
		Obtain $\pi(\mathrm{e}^{p} + 4\mathrm{e}^{\frac{1}{2}p} + p - 5)$	**A1** **8**	**AG**; necessary detail required
	(ii)	State or imply $\dfrac{\mathrm{d}p}{\mathrm{d}t} = 0.2$	**B1**	maybe implied by use of 0.2 in product
		Obtain $\pi(\mathrm{e}^{p} + 2\mathrm{e}^{\frac{1}{2}p} + 1)$ as derivative of V	**B1**	
		Attempt multiplication of values or expressions for $\dfrac{\mathrm{d}p}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}V}{\mathrm{d}p}$	**M1**	
		Obtain $0.2\pi(\mathrm{e}^{4} + 2\mathrm{e}^{2} + 1)$	**A1√**	following their $\dfrac{\mathrm{d}V}{\mathrm{d}p}$ expression
		Obtain 44	**A1** **5**	or greater accuracy